International Commission on Mathematical Instruction
http://www.mathunion.org/ICMI

The 23 ICMI Study:

Primary Mathematics Study on Whole Numbers

3.2. WHOLE NUMBER THINKING, LEARNING, AND DEVELOPMENT

COORDINATORS: J. MULLIGAN & L. VERSCHAFFEL

WG2: Participants

- Joanne Mulligan (coordinator)
- Lieven Verschaffel (coordinator)
- Marja Van den Heuvel-Panhuizen (only presenter)
- Andreas Obersteiner
- Der-Ching Yang
- Shengqing He
- Yunpeng Ma / Sun Xie / Yan-Lin Wang
- Anna Baccaglini-Frank
- Nathalie Sinclair / Alf Coles
- Peter Gould
- Nicky Roberts
- Jasmina Milinkovic

General theme of WG2

- Cognitive (developmental) and neurocognitive analyses of WNA learning, in relation to traditional, present and possible future practices in the early teaching and learning of WNA.
  - The development of foundational core aspects of number sense, in relation to (later) WNA learning
  - The contribution of (neuro-)cognitive studies to our understanding of the learning of WNA in children?
  - What is the importance of the body and bodily experiences in children's learning of WNA
  - The development of children's representations of and strategies for WNA

WG2 activities

- Presentation of the papers

- Discussion of the essential “take-home message” of each paper (homework slide 1) and its broader implications for WNA (homework slide 2)

- Brief discussion of key words that dominated the WG2 presentations and discussions and first attempt to generate a structure for the book chapter

Key words

- Task / tool design (representations of tasks)
- Cardinality vs ordinality of number
- School culture / cross-cultural differences
- Individual differences
- Neuroscientific findings
- Reasoning /patterns & structure / relations (e.g. part-whole, inversion)
- Visual / haptic / movement
- Developmental pathways
- Exceptionality / dyscalulia
- Children's representations of number and of arithmetic operations
- Methodological issues
- Embodied cognition
- Argumentation / justification / discussion / explanation
- Core foundations of WNA (the "starter's kit for learling WNA)
- Instructional issues / national standards / assessment
- The mental number line
- Variation, efficiency and flexibility of arithmetic strategies use
- Connections between concepts / connectivity (see patterns &structure)
- Spatial approaches to WNA
- Development / assessment of WNA competencies

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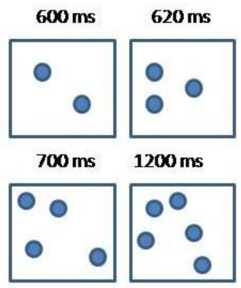
Some (tentative) decisions

- Acknowledgment of realizations and promises of neuroscienific research, combined with a critical look from a math ed perspective
- Focus on thinking, learning and development (without neglecting the socio-cultural and educational setting)
- Attention to intercultural and individual differences (but not to children with special needs)
- Attention to task design (esp. for diagnostic and analytic purposes) and to research methodology
- Focus on children's "starter's kit" and how it relates to their later WNA learning

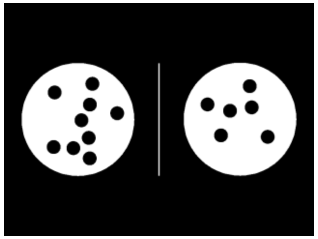
1. State of the art in cognitive neuroscience (see B. Butterworth's plenary lecture)

- From a very young age, humans have a an (inherited) core capacity for numerical processing (e.g., subitizing, representing non-symbolic numerical magnitudes (on a mental number line)
- Symbolic representations (3, 100, ½, 3.17 …) are gradually mapped onto these non-symbolic representations
- These magnitude representations are commonly assessed via subitizing, magnitude comparison and number line estimation tasks

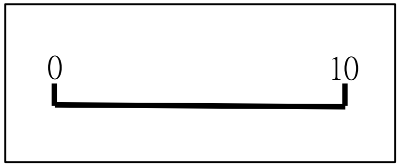
How many



Which set is the largest ?



Where is … 6 ?



State of the art in cognitive neuroscience (see B. Butterworth's plenary lecture)

- Numerical magnitude understanding is positively related to (general) mathematics achievement

- Numerical magnitude understanding can be improved by means of game-based intervention programs

Broadening the realm of foundational core competences for WNA

- Current cognitive neuropsychological research is focused on the development of (the cardinal aspect of) number understanding and its importance for later mathematical development
- Our chapter will broaden the realm of *early math-related competencies*
  - Ordinal aspect of number
  - Measurement aspect of number
  - Competencies related *mathematical relations (commutativity, inversion…)*
  - Competencies related to *structure sense* (cf. awareness of patterns and structure)

Development of representations and strategies in WNA

- In most current cognitive (neuroscientific) studies, children’s foundational competencies are related to children’s **general** mathematical achievement (as measured by standard school achievement test)
- Several papers in our WG2 analyzed the development of more specific aspects of WNA (sometimes in relation to certain aspects of children’s “starter’s kit”), such as the variation, efficiency and flexibility of representations and strategies for
  - counting
  - mental arithmetic
  - written arithmetic
  - computational estimation

Unraveling the development of WNA and its foundations: Methodological issues

- Critical reflection upon the tasks being used in cognitive neuroscientific research from a math educational perspective (e.g. number line task as a "pure" measure of children's underlying numerical magnitude representation)

- Importance of longitudinal and intervention studies, to adequately trace children's development, to get better insight into the causal relations between foundational math-related related abilities and WNA learnt at school, and to come up with evidence-based instructional tasks, tools and techniques